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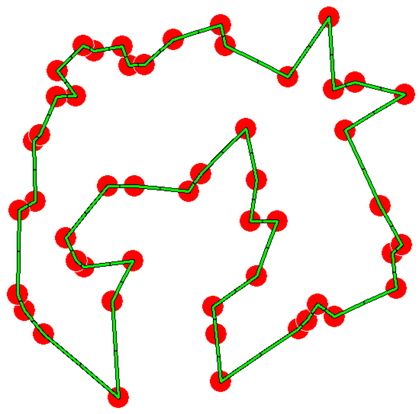
Traveling Salesman Problem
The Role of Clustering Operations

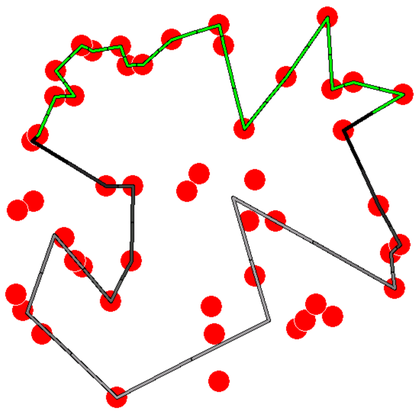
PURDUE UNIVERSITY

Traveling Salesman Problem on the Euclidean Plane

50-city problem and its optimal solution

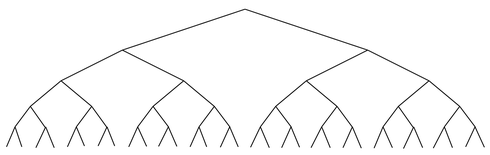


Snapshot of the pyramid model's solution process

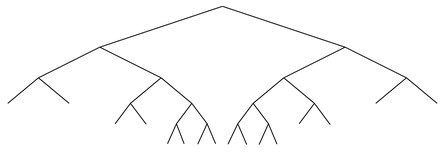


Pyramid Architecture: Conventional vs. Foveating

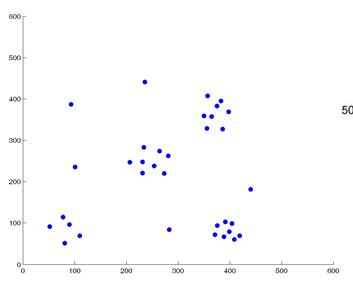
Conventional (Full) Pyramid

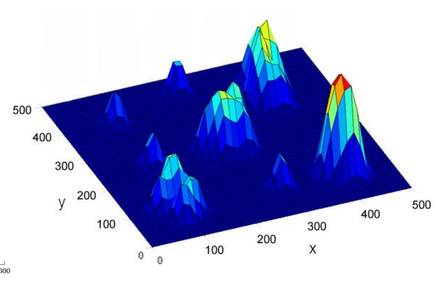


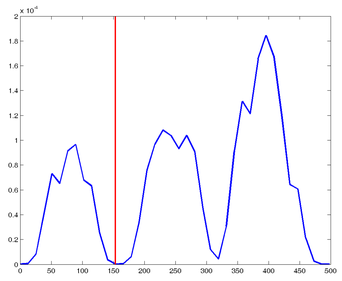
Foveating Pyramid: Simulates the human visual system

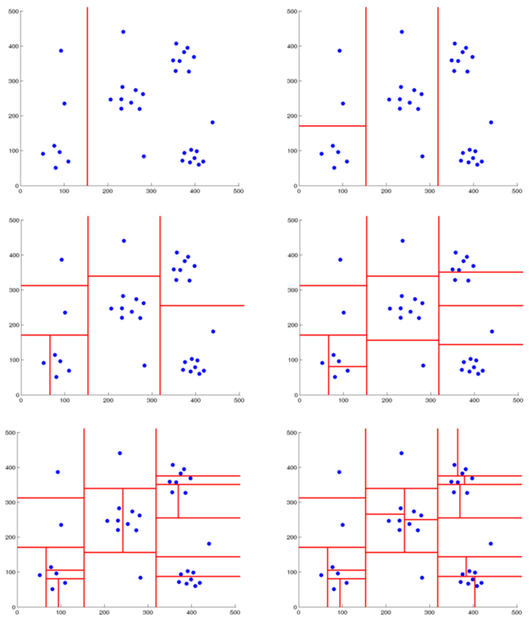


Hierarchical Clustering of Cities: Blurring Followed by Min-Max Cuts







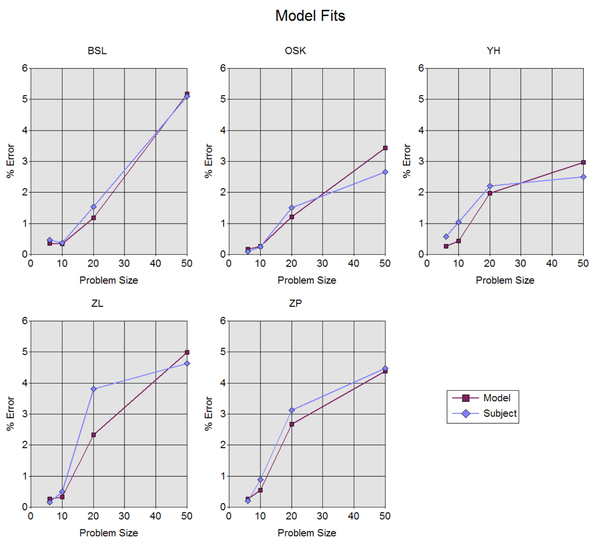


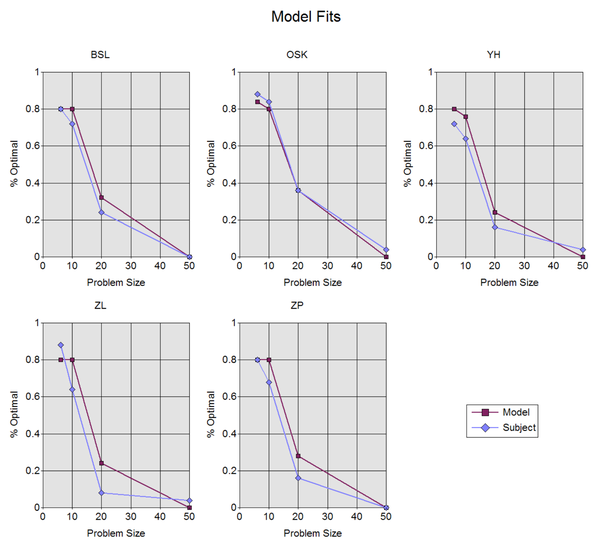
A TSP problem (top left) is blurred using a Gaussian filter. The resulting intensity distribution is shown in the top center. Peaks of the intensity distribution correspond to clusters of cities. The boundaries between clusters are determined (recursively) by finding the minimum from the maximum intensity along x and y directions.

The six images to the left show the series of cuts made on successive levels of the pyramid. In the first image, only one cut has been made, and two clusters have been identified. In the second image, each of these clusters is divided into two smaller clusters. This proceeds recursively until the entire problem has been divided up into a pyramid of clusters, with large, low-resolution clusters at the top of the pyramid, and small, high-resolutions clusters at the bottom of the pyramid.

The solution is produced in a coarse-to-fine process of successive approximations of a TSP tour. The model's performance is compared to human performance (see below). Go to http://psych.purdue.edu/tsp/files/animations/Sample_BisecionPyr_50City.htm to see an example of this process.

Psychophysical Results and the Model's Fits

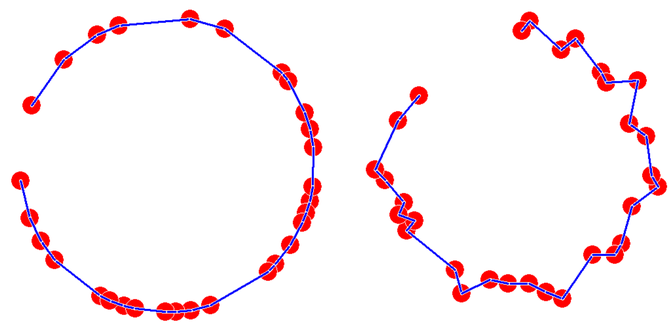




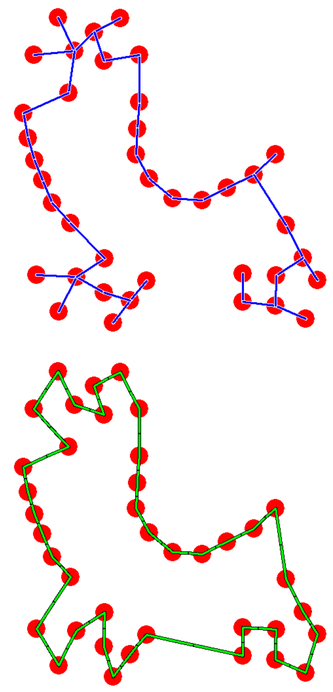
The Role of "Line Clusters"

Minimum Spanning Tree as a Line Detector

Simple Minimum Spanning Trees



A Minimum Spanning Tree and Optimal TSP Solution



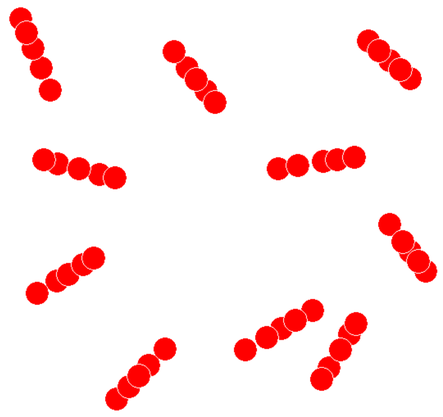
In the two problems shown above, the arrangement of cities is not random. In the first case all cities lie on a circle. In the second, they lie within a short distance of a circle. Both problems are trivial. In both cases, the minimum spanning tree is *non-branching* and specifies the order of cities in the TSP tour.

In the problem shown on top right, there are also non-branching sections of the minimum spanning tree, which correspond to the sections of the problem in which the cities form lines. It follows that the minimum spanning tree can serve as a line detector.

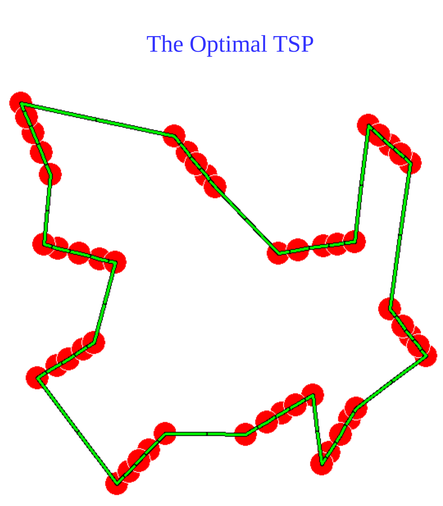
Psychophysics

Three subjects (the authors) were tested with stimuli as shown below. The number of lines was: 4, 6, 8, 10 and 12. There were 5 cities per line, 25 randomly generated problems per each problem size. The results show that the current model which detects blob clusters only, cannot account fully for the data.

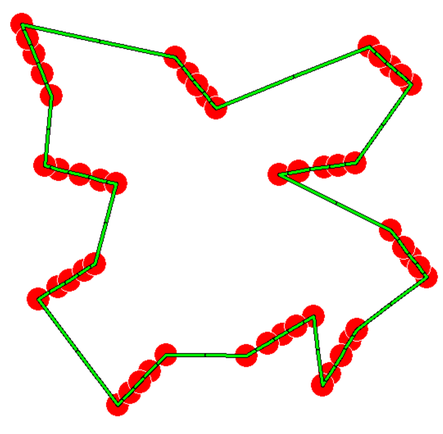
A 10 line Problem

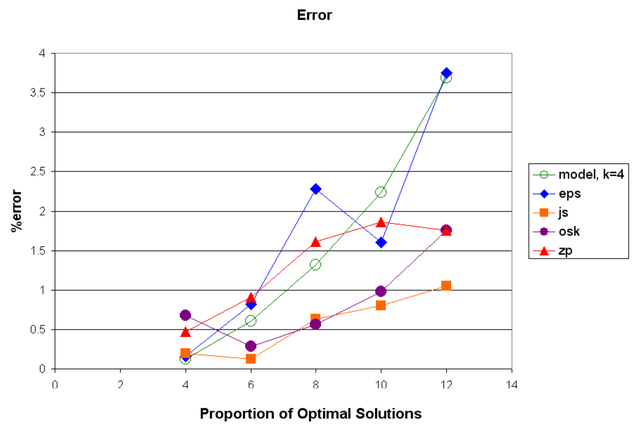


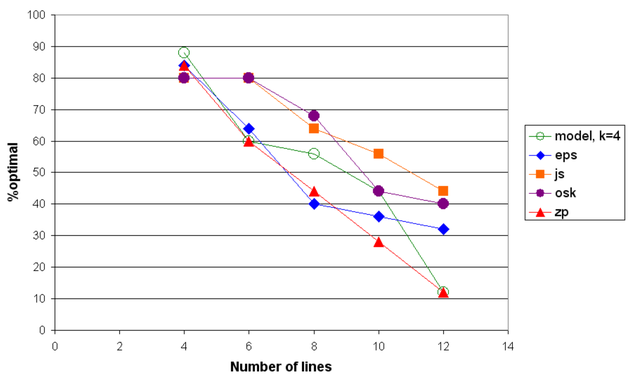
The Optimal TSP



EPS's Solution







Line patterns are more difficult for the model than random distribution of cities. This is not the case with subjects.